Asymptotic properties of restricted naming games

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Abstract

The naming game (NG) is a simple algorithm that aims to describe how a consensus name of any arbitrary object or entity evolves within a large community of agents [1, 2, 3]. More specifically, when a new object is introduced to a community, different agents refer to it by different names. At the same time, the agents mix among themselves and share information. This sharing process is considered as the interaction among the agents. Under this interactive dynamics, gradually different distinct names disappear. Finally, after a long time, only one name becomes most popular and therefore all agents refer to the object by this common name. The NG models present in the literature assumes an infinite capacity of memory of individuals. However, in reality, an individual agent has only a finite amount of memory. In this work, Asymptotic properties of the symmetric [4] and asymmetric naming games have been studied under some restrictions in a community of agents. In one version, the vocabulary sizes of the agents are restricted to finite capacities. In this case, compared to the original naming games, the dynamics takes much longer time for achieving the consensus. In the second version, the symmetric game starts with a limited number of distinct names distributed among the agents. Three different quantities are measured for a quantitative comparison, namely, the maximum value of the total number of names in the community, the time at which the community attains the maximal number of names, and the global convergence time. Using an extensive numerical study, the entire set of three power law exponents characterizing these quantities are estimated for both the versions which are observed to be distinctly different from their counter parts of the original naming games.

Introduction and Motivation

- Names of an object vary over space.



Chalk	Apple	Hammer
Khariya	Sev	Hathora
Khori	Aapel	Haturi
CaCO3	Malus domestica	Lever
Chalk	Apple	Hammer

Consensus among names.

- Physical, Electronic etc. memories has practical limitations.
- Naming game models ⟶ Infinite memory.

⇒ How consensus among names emerges with finite resources.
⇒ Effect of finite memory.

Model: Limited vocabulary naming game

- N agents in a community.
- Every agent has its own vocabulary of length $\ell_i = 1$.
- Vocabulary: A memory where a list of distinct names of an object are kept.
- Each vocabulary contains a distinct word initially.
- The community has N distinct words to start with.
- Vocabulary length can increase upto a cut-off length s, same for all agents.
- Dynamics evolves under mutual pairwise interactions.
  - Two agents are selected randomly at every interaction step.
  - The selected agents i and j interacts with each other.
  - Possible Interactions:
    - ⇒ Success.
    - ⇒ Failure.

The Interaction Rules: Success

- The selected pair of agents compare the words in their inventories.
- Success: If there is at least one common word its a successful move. Mathematically it satisfies: $\{\ell_i(t-1)\} \cap \{\ell_j(t-1)\} \neq \varnothing$
- The common words are kept and other words are deleted.

Agent-i	Agent-j		Agent-i	Agent-j
Book	Apple		**Book**	**Book**
Pencil	**Tree**	⟹	**Tree**	**Tree**
Tree	Rat			
	Book			

- After a successful interaction, new inventory lengths of both the agents vocabularies get modified as: $\{\ell_i(t)\} = \{\ell_j(t)\} = \{\ell_i(t-1)\} \cap \{\ell_j(t-1)\}$
- The cut-off length, s, has no role in successful move.

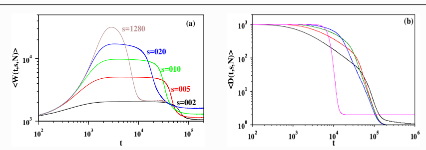
The Interaction Rules: Failure

- There are no common words in their inventories.
- Both the agents share the common shared inventory after interaction.
- If the shared inventory length surpasses cut-off length s, only s words are kept.
- Interaction rule for cut-off length, $s = 5$:

Initial List of words in the vocabularies			List of words after interaction without cut off			List of words after Interaction with cut off	
Agent-i	Agent-j		Agent-i	Agent-j		Agent-i	Agent-j
Rat	Milk		Rat	Rat		Rat	Rat
Paper	Shoe	⟹	Paper	Paper	⟹	Paper	Paper
	Moon		Milk	Milk		Milk	Milk
			Shoe	Shoe		Shoe	Shoe
			Moon	**Moon**			

⇒ Inventory length of Agent-i ⇒ l_i
⇒ Inventory length of Agent-j ⇒ l_j
⇒ After interaction new inventory length of both agents modifies as:
$l_i(t) = l_j(t) = min(s, l_i(t-1) + l_j(t-1))$

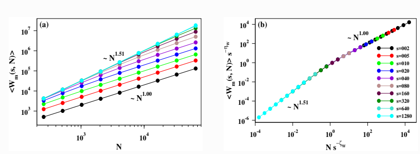
Observables of the Game



Fig(a): The Total number of words $\langle W(t,s,N)\rangle$ has been plotted against time for different s values.
Fig(b): The number of distinct words$\langle D(t,s,N)\rangle$ has been plotted against time for different s values.

- The total number of words $W(t,s,N)$:
  - Large s: Increases upto a maximum (peak) $W_m(t,s,N)$ at $t_m(s,N)$ and then decays to a consensus with one or two words at $t_f(s,N)$.
  - Large s: Increases upto maximum $W_m(t,s,N)$ at t_1 then fluctuates around $W_m(t,s,N)$ upto t_2 and then decays to a consensus.
  - In this case $t_m(s,N)$ is defined as $[t_1(s,N) + t_2(s,N)]/2$
- The number of distinct words $D(t,s,N)$:
  - Behaviour: Remains saturated for some waiting time at $D = N$ before decaying to reach a consensus.
  - Large s: Longer waiting time, sharper decay and smaller t_f.
  - Small s: Little waiting time, slow decay and comparatively larger t_f.
- Observables:
  - $\langle W_m(s,N)\rangle$: Average maximum number of words.
  - $\langle t_m(s,N)\rangle$: Average maximal time.
  - $\langle t_f(s,N)\rangle$: Average Consensus time.

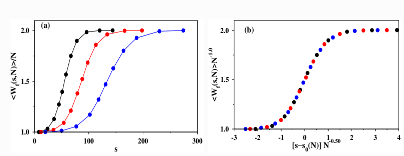
Average maximum number of words



Fig(a): The $\langle W_m(s,N)\rangle$ has been plotted against s for different system sizes (N).
Fig(b): Replot of the transformed axes $\langle W_m(s,N)\rangle s^{-\eta_W}$ and $Ns^{-\zeta_W}$, showing a data collapse for $\eta_W = 3.0$ and $\zeta_W = 2.0$.

- $\langle W_m(s,N)\rangle \sim N^{\gamma(s)}$.
- Smaller s limit: $\gamma(s) \sim 1.00(1)$.
- Larger s limit: $\gamma(s) \sim 1.539(1)$.
- The crossover system size grows as $N_c(s) \sim s^2$.

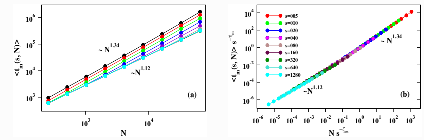
The converged state



Fig(a): The average number of words at the converged state $\langle W_f(s,N)\rangle$ has been plotted against s for different system sizes N = 512 (black), 1024 (red), and 2048 (blue).
Fig(b): Data collapse for the finite size scaling of $\langle W_f(s,N)\rangle/N$ against $[s - s_0(N)]N^{-0.5}$.

- $\langle W_f(s,N)\rangle$ is the average number of words at the converged state.
- The system converges to either $W_f^1(s,N) = 1$ or $W_f^2(s,N) = 2$.
- From smaller s limit the $\langle W_f(s,N)\rangle$ grows with increasing s values.
- $s_0(N)$ is the value of s for which $\langle W_f^1(s,N)\rangle = \langle W_f^2(s,N)\rangle = 2$
- $s_0(N)$ is found to grow as $N^{0.64}$.

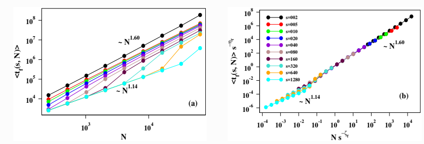
Average maximal time



Fig(a): The $\langle t_m(s,N)\rangle$ has been plotted against s for different system sizes (N).
Fig(b): Replot of the transformed axes $\langle t_m(s,N)\rangle s^{-\eta_m}$ and $Ns^{-\zeta_m}$, showing a data collapse for $\eta_m = 3.0$ and $\zeta_m = 2.53$.

- $\langle t_m(s,N)\rangle \sim N^{\alpha(s)}$.
- Smaller s limit: $\alpha(s) \sim 1.34(1)$.
- Larger s limit: $\alpha(s) \sim 1.12(1)$.

Average convergence time



Fig(a): The $\langle t_f(s,N)\rangle$ has been plotted against s for different system sizes (N).
Fig(b): Replot of the transformed axes $\langle t_f(s,N)\rangle s^{-\eta_f}$ and $Ns^{-\zeta_f}$, showing a data collapse for $\eta_f = 3.0$ and $\zeta_f = 2.0$.

- $\langle t_f(s,N)\rangle \sim N^{\beta(s)}$.
- Smaller s limit: $\beta(s) \sim 1.60(1)$.
- Larger s limit: $\beta(s) \sim 1.14(1)$.

Limited number of distinct names

- The number of distinct names are limited to $n = N^\delta$.
- Each distinct name is assigned to a set of int$[N/n]$ agents initially.

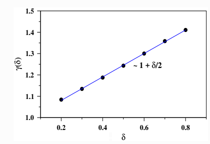


Fig: The characteristic exponent $\gamma(\delta)$ has been plotted as a function of δ for $N = 2^{11}$.

- The characteristic exponent γ behaves as: $\gamma(\delta) \sim 1 + \delta/2$
- The other two exponents $\alpha(\delta)$ and $\beta(\delta)$ also depends on δ.
- Both these initially grows from their counter part of smaller s values, reaches a peak around $\delta \sim 0.65$ and then decays to the corresponding values of larger s limit.

Comparison and Conclusion

Naming Game	α	β	γ
Asymmetric game[1,2,3]	1.5	1.5	1.5
Symmetric game[4]	1.12	1.14	1.539
Rest. vocabulary (Symm.)	1.34(1)	1.60(1)	1.00(1)
Rest. vocabulary (Asymm.)	1.87(1)	2.18(1)	1.50(1)
Rest. names	1.09(1)	1.10(1)	0.99(1)

1. Comparisons of different NG models:
   - Asymmetric NG: Times scales are faster with $\sim N^{1.5}$ order of memory.
   - Symmetric NG: Times scales are fastest with the same ($\sim N^{1.5}$) order of memory.
   - Restricted naming games has limited memory that leads to comparatively slower convergence, however are more realistic.
2. Restricted memory naming games:
   - Limited memory.
   - Tunable memory parameter.
   - Crossover from a restricted behaviour to a unrestricted behaviour with tuning vocabulary size.

References

1. *A. Baronchelli, L. Dall'Asta, A. Barrat and V. Loreto, Phys. Rev. E 73, 015102R (2006).*
2. *L. Dall'Asta, A. Baronchelli, A. Barrat and V. Loreto, Phys. Rev. E 74, 036105 (2006).*
3. *A. Baronchelli, M. Felici, V. Loreto, E. Caglioti and L. Steels, J. Stat. Mech. (2006) P06014.*
4. *B. Bhattacherjee, A. Mukherjee, S. S. Manna, Phys. Rev. E 87, 062808 (2013).*